

Stationary Queue Length Distribution for a Discrete-Time Preemptive Priority Queue

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Abstract

This paper considers a discrete-time Geo/Geo/1 preemptive priority queue with two classes of customers. The queue system can be modeled by a quasi-birth-and-death (QBD) process with infinitely many phases. For the QBD process, we obtain explicit form of the joint stationary distribution and then we also get the stationary queue length distribution for lower-priority customers.

Keywords: Geo/Geo/1 queue, preemptive priority, QBD process, joint stationary distribution, stationary queue length.

1. Introduction

During the last two decades, discrete-time queue models have been well investigated due to their potential application to various areas such as computer system, control system, communication system and so forth. Since such systems are more digital than analogue, we work in discrete-time model has become more appropriate. Pioneering work on discrete-time queue model was given by Meisling [10]. And a comprehensive review of various models, methods and results can be found in either the expository paper of Alfa [2], or the monographs of Alfa [4, 5].

Moreover, queueing models with priority discipline are becoming a hot topic, since such models can depict the fact that different customers may have different priority. For example, Vázquez [15] had used priority queue for modeling bursts and heavy tails in human dynamics. There were many studies on priority queueing models. Cobham [6] was the first to consider the non-preemptive priority queue. Pioneering work on preemptive queues was given by White [16]. Miller [11] firstly considered the M/M/1 preemptive and non-preemptive priority queues by quasi-birth-and-death (QBD) process with infinite phases. By extending Neuts' matrix-analytic theory (see Neuts [12]) to blocks with infinite size, Miller exploited the upper triangular structure of rate operator and developed a recursive computational formula for the steady-state probabilities. Subsequently, some researchers extended Miller's method to other more general models. In