

## **An M/G/1 Queue with Additional Second Stage Service and Optional Re-Service**

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### **Abstract**

We study a single server queue with Poisson arrivals, two stages of heterogeneous service with different general (arbitrary) service time distributions. The first stage service is compulsory whereas the second stage is optional. All arriving customers take the first stage of services after completion of which a customer has the option to repeat or not to repeat the first stage service and leave the system without taking the second stage or take the second stage service. Similarly after the second stage service he has yet another option to repeat or not to repeat the second stage service. We obtain steady state results in explicit and closed form in terms of the probability generating functions for the number of customers in the queue and the system, the average number of customers and the average waiting time in the queue as well as the system. Some special cases of interest are discussed and some known results have been derived. A numerical illustration is provided.

*Keywords:* Poisson Arrivals, Two-Stage Heterogeneous Service, Optional Re-Service, Steady State, Probability Generating Function.