

Characterizations of the Uniform Distribution by Conditional Expectations

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Abstract

Let X be a random variable on the interval $[a, b]$ with continuous distribution function F . If $E(X|X > c) = \frac{b+c}{2}$ for $a < c < b$, then X has a uniform distribution on $[a, b]$. Also let $X_{1:n} < X_{2:n} < \dots < X_{n:n}$ be the order statistics of a random sample of size n from F , the relation $E(X_{k+1:n} - X_{k:n} | X_{k:n} = c) = \frac{b-c}{n-k+1}$, for any $1 \leq k < n$, and $a < c < b$, characterizes the uniform distribution.

Keywords: Uniform Distribution; Characterization; Conditional Expectation; Order Statistics.