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Probability Density Estimation by Ultraspherical Series

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Abstract

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent by identically distributed random variables with an unknown density function $f(x)$. We consider the estimate $f_n(x) = \sum_{k=0}^{l(n)} \hat{a}_{kn}\psi_k(x)$ with $\hat{a}_{kn} = \frac{1}{n}\sum_{j=0}^{n}\psi_k(X_j)\cdot w(X_j)$ where $\psi_k(\cdot)$ is the k th normalized *Ultraspherical* function with weight function $w(x) = (1-x^2)^{(\lambda-\frac{1}{2})}$, $\lambda > -\frac{1}{2}$ and $l(n)$ is an integer depending on sample size n. Under some conditions of $f(x)$.we show consistency of $F_n(x)$ in the sense of mean integrated square error and mean square error. The rate of convergence is also investigated.

Keywords: Probability Density Estimation, Ultraspherical Function, Mean Integrated Square Error, Mean Square Error, Rate of Convergence.